December 8th, 2006

Math 455

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Fall 2006

Name: ....

Student ID (middle 5 digits): XX XX

Final (In-class part)

You have two hours to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the middle five digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 6 questions and 8 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! Even correct answers without work may result in point deductions. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	7	
2	6	
3	7	
4	7	
5	6	
6	7	
Total	40	

1) Give the conjugacy classes and the class equation for Q_8. [**Hint:** Let Q_8 act on itself by *conjugation*. Then the conjugacy classes are the distinct orbits, and the class equation is given by the orders of these classes. The class equation is something like: "$8 = 1 + 1 + 1 + 2 + 3$".]

2) Let R be a ring [with identity, as usual]. Prove that $R^\times$, with the operation of multiplication, is a group.

3) Let R be a ring. An element $a \in R$ is a *zero-divisor* if $a \neq 0_R$ and there exists $b \neq 0_R$ in R such that $a \cdot b = 0_R$. Prove that if R is a *field* [i.e., $1_R \neq 0_R$, and every element but zero has a *multiplicative* inverse], then it has no zero divisors. [Note that, by definition, 0_R is *not* a zero divisor.]

4) Prove that the dihedral group D_{2n} [for $n \geq 3$] is never simple.

5) Let $G \stackrel{\text{def}}{=} \langle x, y, z \ : \ yxyz^{-2} = 1 \rangle$. Prove that $G = \langle y, z \rangle$, i.e., that G can be generated by y and z only.

6) Prove that if $|G| = 8$ and $|G'| = 25$, then the only homomorphism $\phi : G \to G'$ is the one that takes every element of G to the identity of G'.

Scratch: