September 30th, 2008

Math 300

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Fall 2007

Name: .....

Student ID (last 6 digits): XXX- .....

Midterm 1

You have 75 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 8 questions and 7 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! Even correct answers without work may result in point deductions. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	10	
2	10	
3	10	
4	15	
5	10	
6	15	
7	10	
8	20	
Total	100	

1) [10 points] Let $X = \{1, 2, 3\}$. Give an example of a relation $\mathcal{R}$ [not necessarily an equivalence reltaion!] on X such that $1\mathcal{R}2$, but 1 is *not* related to 3 by $\mathcal{R}$.

2) [10 points] Give the definition of *domain* of a relation $\mathcal{R}$ on $X \times Y$.

3) [10 points] Let $X = \{1, 2, 3, 4, 5\}$. Is $\mathcal{P} = \{\{1, 2\}, \{3, 4\}, \{2, 5\}\}$ a partition of X? How about $\mathcal{Q} = \{\{1, 2\}, \{3\}, \{4\}\}$? Justify your answers!

4) [15 points] Let x be a real number. Prove that if x is irrational, then $1/x$ is also irrational.

5) [10 points] Prove or disprove: Let A, B and C be sets. Then, $[(A \setminus B) \setminus C] \cup (B \setminus C) = A \setminus C$.

6) [15 points] Prove or disprove: Let A, B and C be sets. Then, $(A \setminus B) \cap C = (C \setminus B) \cap A$.

7) [10 points] Give the completely simplified negation of the following statement. [Your answers should have no "nots" in them.]

> *Given $x > 0$, there exists y such that (if $x < 1$, then $y < x$) and (if $x \geq 1$, then $y \geq x$).*

[**Hint:** Negate one part at a time, as it makes it easier to get partial credit.]

8) Let $X = \mathbb{R} \setminus \{0\}$. Define a relation $\mathcal{R}$ by $x\mathcal{R}y$ if $x/y \in \mathbb{Q}$.

(a) [15 points] Prove that $\mathcal{R}$ is an equivalence relation.

(b) [5 points] Describe the equivalence class of 1. [Your answer should be a *simple* and well described subset of X. Just writing the definition is not enough, although it'd get you some partial credit.]

Scratch: