Midterm

M551 – Abstract Algebra

October 14th, 2011

Solve as many as you can in class. (I would hope you could do all of this in class, as it is comparable to one part of the prelim.) Then, you may take this sheet home and solve all remaining problems, or problems that you think you've missed, and bring it to class on Monday. I will consider it for some partial credit. [At *most* half of the original number of points you've missed in the question.]

You should treat these problem as a *take-home* exam, not as a homework. So, you should not discuss *anything* about these problems with *anyone*. You can, however, use your book and notes. (No internet or other textbooks, though.)

1. [40 points] Let G be a finite *simple* group with $|G| = p^{\alpha} n$, where $n > 1$, p is prime, $\alpha \in \mathbb{Z}_{>0}$, $n > 1$, and $p \nmid n$. Show that $|G| \mid n_p!$, where n_p is the number of Sylow p-subgroups of G.

2. (a) [30 points] Let G act on a finite set S and assume that there exists an element in G which induces an *odd* permutation of S. Show that there exists $H \leq G$ such that $|G : H| = 2$.

(b) [30 points] Let G be a finite group of order $2n$, where n is odd. Show that G has a subgroup of index 2. [**Hint:** Let G act on itself by left multiplication.]