Exam 2

You must upload the solutions to this exam by 11:59pm on *Sunday* 07/30. [Note it was postponed!] Since this is a take home, I want all your solutions to be neat and well written.

You can look at class discussions on Cocalc and *our* book only (*except* for the hints to exercises in the back of the book)! You *cannot* look at our videos, solutions posted by me or *any* other references (including the Internet) without my previous approval. Also, of course, you cannot discuss this with *anyone*!

You can use a computer only to check your answers, as **you need to show work in all questions**.

Note: In all "True or False" questions, you need to justify your answer. [Usually a proof if True and a counter-example if False.]

1) [16 points] Find all the units of $\mathbb{I}_{15}$ and for each unit, find its inverse.

2) [16 points] Prove that the only subring of $\mathbb{I}_m$ is $\mathbb{I}_m$ itself.

3) [20 points] True or False:

(a) A subring of a field is always a field.

(b) A subring of a field is always a domain.

4) [16 points] True or False: If F is a field, then there is a domain R with $R \subseteq F$ and $R \neq F$ such that $F = \text{Frac}(R)$.

[**Note:** Remember that $\text{Frac}(R)$ denotes the field of fractions of R. Note also that it is important here that $R \neq F$, for we always have that if F is a field, then $\text{Frac}(F) = F$.]

5) [16 points] Simplify:

(a) $([1] + [4]x)^3$ in $\mathbb{I}_8[x]$.

(b) $([1]x^2 + [1]x^3 + [1]x^5)^2$ in $\mathbb{I}_2[x]$.

(c) $([2]x + [1]x^4)^3$ in $\mathbb{I}_3[x]$

6) [16 points] Let R be a commutative ring. Prove that $R[x]$ is never a field.